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NON-FOURIER COMPUTER GENERATED HOLOGRAPHY FOR 3-D DISPLAY

THESIS

Tommy A. Mouser
Captain, USAF

AFIT/GCS/ENG/89D-13

DEPARTMENT OF THE AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Computer Science

Tommy A. Mouser, B.S.
Captain, USAF

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Preface

The purpose of this thesis effort was to begin a research effort into the development of Computer Generated Holography. This involved the development of a system capable of producing a computer generated hologram.

I am indebted to several people for their assistance on this project. First, I would like to thank Major Phil Amburn, my thesis advisor, for his constant encouragement, support, and advice. Second, I would like to thank Dr. Matt Kabrisky and Major Steve Rogers who served on my thesis committee. Their initial efforts to bring me up to speed on holography are greatly appreciated. Next I would like to thank Captain Sean Kelly, AFSC WRDC/KTD, for his contribution of technical information.

Last, I wish to thank my wife Debbie, and my children Karen and Benjamin for their constant love and encouragement during the last 18 months.

Finally, since this thesis effort deals fundamentally with light, and since I wish to give credit where credit is due, the following quote is appropriate.

> In the beginning, God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the spirit of God move upon the face of the waters. And God said, Let there be light: and there was light. And God saw the light, that it was good: And God divided the light from the darkness.

Genesis 1:1-4

The *Authorized* Version

Tommy A. Mouser

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Abstract

Computer generated holograms are used for a variety of purposes. One promising approach is to use them to provide three dimensional views of an object. The methods currently in use to develop computer generated holograms use the Fast Fourier Transform and are not geared toward developing three dimensional images.

This thesis effort developed a method to produce a computer generated hologram by implementing an equation almost identical to that of the general form of scalar diffraction theory. This method will theoretically allow computer generated holograms to be made for a wider range of objects.

The interference pattern for a cube was calculated using this method. An electron-beam lithography machine was used to transform this pattern to a glass plate.

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I. Introduction

1.1 Background

Holography is a technique very similar to photography. The primary difference between these two optical recording techniques is the amount of information that is recorded. Both techniques use photographic film to record intensity. With photography only the intensity of light striking the photographic film is recorded. However, in holography the intensity pattern recorded on the film has phase information encoded in it. Because holography records both phase and intensity information, a hologram can be used to produce a duplicate of the wavefront that came from the object recorded in the hologram. For this reason holography can be used to provide true three-dimensional imaging while photography can provide only two-dimensional imaging. The hologram's ability to produce these true three-dimensional images has intrigued people for many years and has prompted much research, including that reported in this thesis.

Holography was developed in 1948 by Gabor in an attempt to reduce the aberrations of electron microscopes (10). Since that time research in the field has equation needed to produce a CGH is

$$H(X_h, Y_h) = \int\int\int \frac{exp[i2\pi\lambda^{-1}((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}]}{((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}} dx_o dy_o dz_o \quad (1)$$

where λ is the wavelength of light, X_o, Y_o, and Z_o are the coordinates of a point on the object, and X_h, Y_h and Z_h are the coordinates of a point on the hologram plane. The integral is taken over the entire surface of the object. The reader unfamiliar with the mathematics of holography is referenced to Appendix A.

A CGH is produced by using equation 1 to compute the intensity value at a finite number of points in the hologram plane. These intensity values are then rounded to either zero or one. The array of zeroes and ones is then contoured and used to drive a plotter. The resulting interference pattern is then photographically reduced to the appropriate size and illuminated with a suitable light source to reconstruct the original image.

Computer generated holography is unique because the object of which a hologram is to be made need not physically exist. Instead only a mathematical description of the object is needed. As a result computer generated holography can be used to produce images of objects that would be very difficult or impossible to produce using standard optical techniques.

1.2 Problem Statement

The purpose of this thesis was threefold. First, answer the following question: Without simplifying equation 1, is the calculation of a CGH a tractable problem using the computer facilities available at the Air Force Institute of Technology (AFIT) and the Air Force Weapons Lab (AFWL) Super Computer Center? Second, develop an initial three-dimensional CGH capability at AFIT. Finally, begin an investigation to determine what non-Fourier simplifications can be made to equation 1 without seriously degrading the quality of the resulting holograms.

1.3 Current Work

Computer generated holography is a well established field (20, 14, 22, 1). Most of the work in this area deals with uses of holography for other than three-dimensional display. Some early research was aimed at three-dimensional display, but little of the research conducted in the last twenty years has dealt with three-dimensional display (18). Instead recent work has dealt with the use of computer generated holography in a variety of other fields such as optical data processing, interferometry, information storage, microscopy, motion pictures, x-ray, and the study of acoustics (17, 23, 7, 13).

1.4 Scope

There exist many methods for calculating the interference pattern of a CGH. Current literature contains a comprehensive discussion of these methods. Chapter II of this thesis provides an overview of the methods found in the literature. The most obvious method of producing a CGH is a two step process. The first step is to use equation 1 to compute the values at a finite number of points in the hologram plane. The second step involves transferring the calculated values in the computer to suitable material to allow illumination and hence the observation of the original object. The computation required to calculate the values in the hologram plane, using this method, was an intractable problem when computer generated holography was first considered.

Because the obvious and straight forward way of producing a CGH is considered an intractable problem research has been directed at ways of simplifying the problem. One simplification which seems to have been universally implemented is to reduce equation 1 into a Fourier integral and then use the Fast Fourier Transform (FFT) algorithm to produce results (26:5). It appears that no effort has been made to simplify equation 1 without converting it into a Fourier integral. Therefore, this thesis effort concentrated on methods of simplification which did not involve Fourier integrals and the FFT.

Regardless of how the interference pattern is calculated the pattern must be transferred from the computer onto a suitable device which will allow illumination of the pattern. There are three primary methods available to transform the calculated values into an actual hologram. Method one is to use the calculated values to drive a spatial light modulator (16). This method has the advantage of being very fast. It has the disadvantage of being temporary because when the spatial light modulator is turned off the hologram is gone. Method two is to use the calculated values to drive a thermal plastic holographic plate (8). This method is slower than method one and has the same disadvantage of being only temporary. Method three is to draw the calculated interference pattern using a plotting device and to then, if necessary, photographically reduce the plot to the appropriate size. Method three is by far the slowest of the three methods, but is unique because it produces a permanent hologram that can be reilluminated and viewed multiple times.

The primary concern of this thesis effort was the calculation of interference patterns in a CGH and not the transformation of the calculated values into an actual hologram. For this reason method three was used exclusively to transform the calculated values into the actual hologram. This allowed multiple holograms produced by different methods to be viewed as necessary during the evaluation phase of the research. Figure 1 provides an illustration of the research conducted for this thesis.

1.5 Approach

This thesis effort was divided into four distinct steps: a literature review, generation of interference pattern software, generation of plotter interface software, and an evaluation phase. Each of these steps is discussed below.

A review of the literature on computer generated holography was conducted. The purpose of the literature review was two fold. First, the literature review established the background necessary to conduct this research effort. Second, the

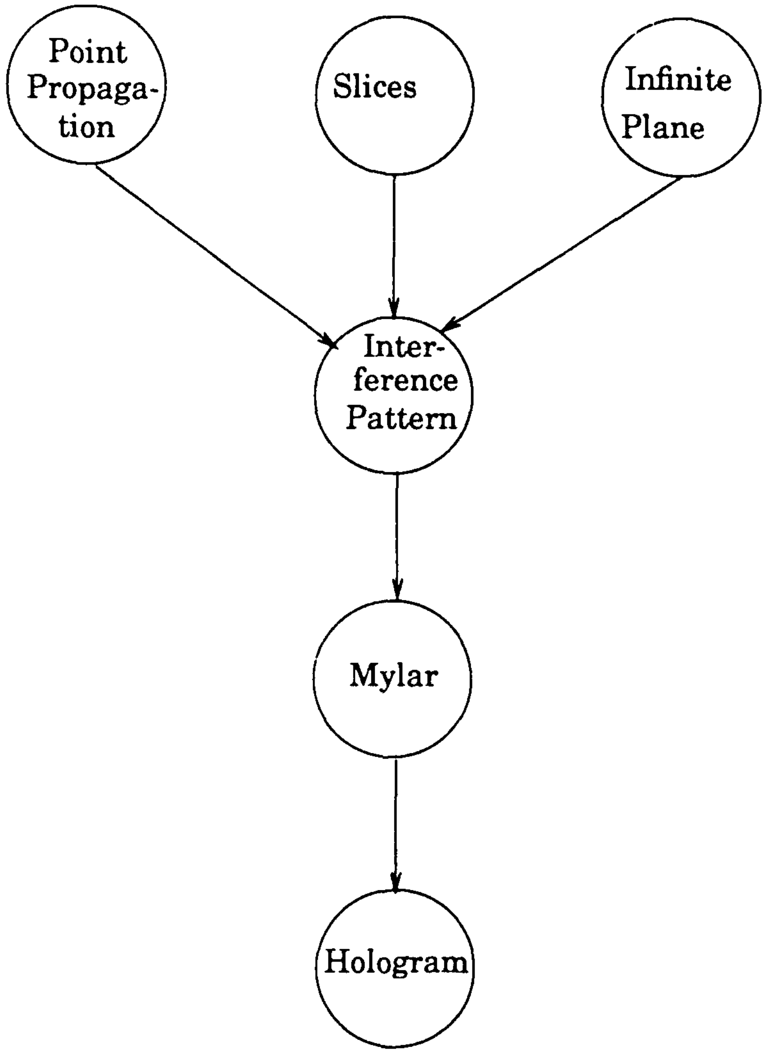

Figure 1. Scope of Thesis

literature review provided an understanding of the current methods available to produce a computer generated hologram. Knowledge of current methods helped to guide the direction of this research.

The second step was the development of interference pattern software. As mentioned earlier there are several methods available to accomplish this task. Simple point propagation using equation 1 was chosen as the first method to implement. This method was chosen first for two reasons. First, it is the method which most accurately simulates the development of a classical optical hologram. As a result, this method is the easiest to understand. The second reason for choosing this method first is that it has been assumed that this method is still an intractable approach to developing CGH. Given the advancement in computer hardware in the past twenty years, a review of this approach was warranted.

The third step was to develop the software necessary to drive the e-beam lithography machine. This step involved becoming familiar with the e-beam input format and developing an algorithm to convert the calculated intensity values into start and stop points for the e-beam lithography machine.

The final step in the process was the evaluation of the holograms produced. This evaluation was based primarily on the aesthetic quality of the images produced. Image brightness and clarity were the major factors considered.

1.6 Equipment Required

Successful completion of this thesis required the following equipment: computers with FORTRAN and C compilers for the calculation of the interference patterns, a plotter to output the calculated interference patterns, and a gas laser to illuminate the hologram. Computers at AFIT and AFWL were used to calculate the intensity values. A plotter capable of writing at the required resolution was not available at AFIT, so the e-beam lithography machine at the Wright Research and Development Center (WRDC) was used. Since the e-beam lithography machine can write at the required resolution, no photographic reduction was required. The gas lasers used to illuminate the holograms were available at AFIT.

II. Current Knowledge

2.1 Introduction

Since 1948, when Gabor introduced holography, the field has grown to include applications in a variety of wavelengths in both the electomagnetic and acoustic ranges (7, 13, 23). Holograms of all wavelengths can be formed digitally and, as a result, their potential was recognized early on and investigation into their use began (18, 4, 24). When CGHs are made using optical wavelengths four distinct steps are carried out (1:291):

1. Choosing the desired object.
2. Calculating the object wave front at the hologram plane.
3. Encoding this wave front into the hologram transmittance.
4. Optically decoding the transmittance by illuminating the hologram with the reference wave.

CGHs were first reported by Brown and Lohmann in 1966 as an attempt to develop a hologram for optical spatial filtering (3). Their success in this area stimulated research to determine applications for CGHs and to determine alternate methods to develop CGHs. An important aspect of computer generated holography was also noted by Brown and Lohmann. The problem of synthesizing a hologram is the opposite from typical diffraction problems. With normal diffraction problems the object is given and the diffracted field is then determined. With computer generated holograms the image is prescribed and the hologram, or diffracting object, is determined.

2.2 Types of Computer Generated Holograms

There many different ways to digitally produce the interference patterns necessary to make a hologram. This section will discuss the different methods. In addition, the work currently being conducted by Steve Benton on synthetic holographic stereograms is discussed.

2.2.1 Actual Computer Generated Holograms The methods available to create true computer generated holograms can be divided into two categories. The first category is point propagation and involves the implementation of equation 1. The second category involves applying two simplifying assumptions to equation 1 and uses Fourier Analysis (26:4-5).

2.2.1.1 Equation 1 Implementation Although equation 1 is the fundamental equation used to mathematically describe the intensity patterns produced in a hologram, there appears to be no published accounts of its direct implementation. Presumably this is due to the fact that development of a CGH using this method is considered to be an intractable problem.

2.2.1.2 Fourier Analysis Methods The methods discussed in this section are all related by the fact that Fourier analysis is used to solve the problem. The use of Fourier analysis is desirable because this allows use of the Fast Fourier Transform (FFT) algorithm. However, before the problem of developing a CGH can be properly synthesized by a Fourier Transform, two simplifying assumptions must be made. First, if the dimensions of the object of which a hologram is to be made are small in comparison with the distance separating the object and the hologram plane, then equation 1 can be reduced to a Fresnel integral. Second, if the hologram plane is small in comparison with the distance separating the object and the hologram plane, then the Fresnel integral resulting from the first assumption can be further reduced into a Fourier integral (26:4-5). Thus, if these two assumptions are true, the use of the

FFT is a valid solution to the integral and a significant time savings is possible. The remainder of the section is devoted to discussion of different approaches to developing CGHs using Fourier analysis. See Appendix B for more detail on the conversion of equation 1 into a Fourier integral.

Cell Oriented Holograms The first Computer Generated Holograms, developed by Brown and Lohmann, were originally called binary holograms (3). This was because each point on the hologram transparency had a value of zero or one. That is, each point was either completely transparent or completely opaque. Brown and Lohmann developed their CGHs by taking the Fourier transform of an image and transferring the transform values to the hologram transparency, or mask. The transform plane was divided into cells of equal size. Brown and Lohmann discussed three methods of representing the data. In the first method each cell contained two separate apertures. The total width of the two apertures depended on the magnitude of the transform value at the center of the cell. In methods two and three each cell had only one aperture whose height and width depended on the magnitude of the transform at the center of the cell. In all three of the methods the lateral position of the aperture was related to the transform's phase at the center of the cell. The name detour phase hologram was later applied to these holograms because this lateral shift in the aperture is similar to diffraction gratings with unequally spaced rulings (22:4353).

Since cell oriented CGHs were introduced by Brown and Lohmann several different variations have been developed. Dallas reports seven distinct types of holograms that can be made using a cell oriented approach (1:295-311). They are all similar and are based on the detour phase holograms developed by Brown and Lohmann. Tricoles provides the following excellent explanation of the equations used for detour phase holograms. Figure 2 explains several of the variables used in the following equations.

The aim is a hologram with transmittance $H(\nu_x, \nu_y)$, which produces an image amplitude $u(x, y)$ when illuminated by a plane wave $\exp(i2\pi\nu_x x_0) = E(\nu_x x_0)$ in the arrangement of figure 3, where $x_H = \lambda f \nu_x$ and $y_h = \lambda f \nu_y$. The requirement is that $h(x, y)$ the amplitude diffracted by the hologram be proportional to that of the image in a finite region; that is,

$$h(x, y) = Ku(x, y). \tag{2}$$

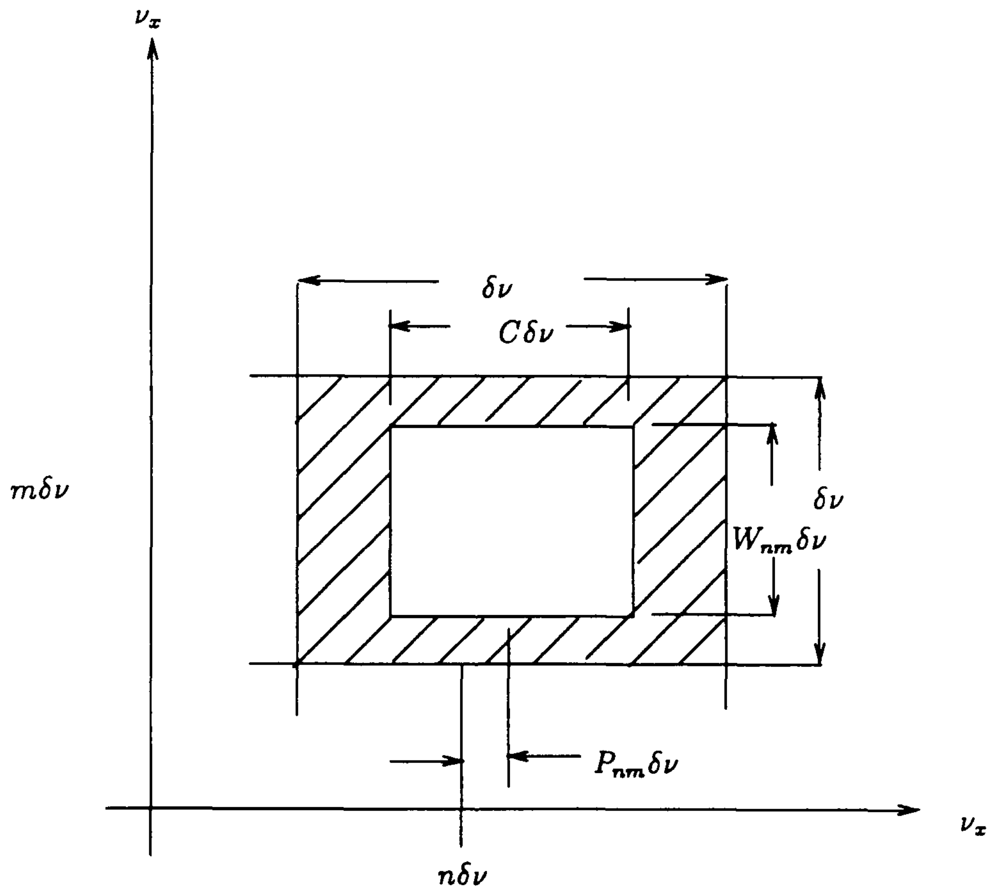

Figure 2. Typical cell in a binary hologram. Position indices are n and m. Width is $C\delta\nu$, height $W_{nm}\delta\nu$,and lateral displacement from the cell center is $P_{nm}\delta\nu$ (22)

With the image width Δx and height Δy, the diffracted amplitude is

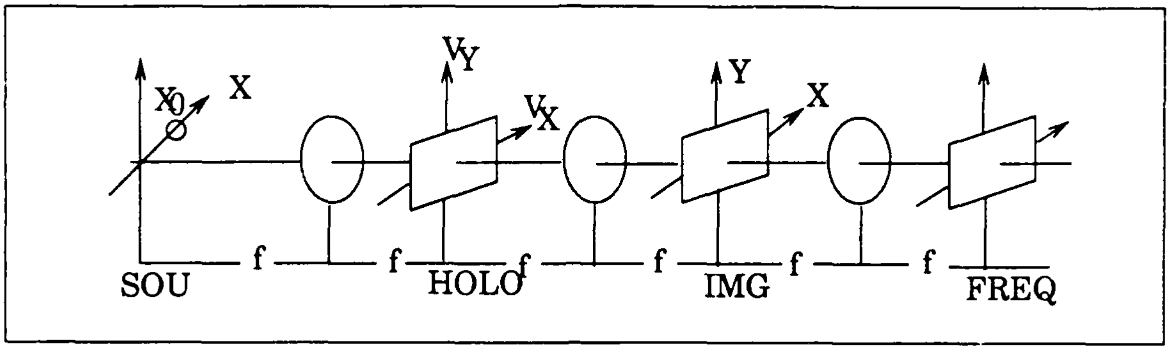

Figure 3. Optical setup for reconstruction with a point source (SOU) at x_0. HOLO, IMG, and FREQ are, respectively, the hologram, image and frequency planes.(22:4353)

$$h(x,y) = rect(x/\Delta x)rect(y/\Delta y)\int\int H(\nu_x,\nu_y)E[(x+x_0)\nu_x + y\nu_y]d\nu_x d\nu_y, \quad (3)$$

where rect$z$ has a unit value for $|z| \leq 1/2$ and zero otherwise.

The hologram is represented by sampled values. The number of samples depends on image size and image resolution. If δx is the resolution element size in the image, the number of resolvable points in the image is, with $\delta_x = (\Delta\nu)^{-1}$,

$$N^2 = (\Delta_x\Delta_y)/(\delta_x)^2 = (\Delta_x\Delta_y)(\Delta\nu)^2. \quad (4)$$

N is called the number of degrees of freedom. The hologram is assumed to have at least N^2 points to preserve information; thus

$$(\Delta\nu/\delta\nu)^2 \geq (\Delta x/\delta x)^2 = (\Delta x\Delta\nu)^2. \quad (5)$$

In fact, $\delta\nu$ was taken to be $(\Delta x)^{-1}$ on the basis of the sampling theorem.

To develop expressions for aperture dimensions, consider equation (2) and apply scalar diffraction theory to the left-hand side. The hologram's transmittance is

$$H(\nu_x\nu_y) = \sum_n\sum_m rect[\nu_x - (n+P_{nm})\delta\nu]/c\delta\nu \times rect[(\nu_y - m\delta\nu)/W_{nm}\delta\nu]. \quad (6)$$

The rectangle functions describe the aperture dimensions in a cell as in figure 3. Illumination by a plane wave produces complex amplitude

$$\begin{aligned} h &= \int\int H(\nu_x \nu_y) E[(x + x_0)\nu_x + y\nu_y] d\nu_x d\nu_y && (7) \\ &= c(\delta\nu)^2 sinc[c\delta\nu(x + x_0)] \sum\sum W_{nm} \times \\ & \quad sinc(yW_{nm}\delta\nu) E\{\delta\nu[(x + x_0)(n + P_{nm}) + ym]\} && (8) \end{aligned}$$

For the right-hand side of equation (2) the described image $u(x, y)$ in its Fourier representation with the tilde signifying the transform is

$$u(x, y) = \int\int \tilde{u}\ (\nu_x \nu_y) E(x\nu_x + y\nu_y) d\nu_x d\nu_y. \qquad (9)$$

Consider the Fourier sampling theorem

$$\tilde{u} = \sum_n \sum_m \tilde{u}\ (n/\Delta x, m/\Delta x) \times sinc(\nu_x \Delta x - n) sinc(\nu_y \Delta x - m) \qquad (10)$$

With equation (10), equation (9) gives

$$u(x, y) = rect(x/\Delta x) \times rect(y/\Delta x) \sum_n \sum_m u(n\delta\nu, m\delta) E[\delta\nu(xn + ym)]. \qquad (11)$$

To satisfy the condition in equation (2), equation (8) is approximated to equal equation (11) term by term. The result is

$$(\delta\nu)^2 WnmE[x_0 \delta\nu(n + P_{nm})] \approx const.\ \tilde{u}\ (n\delta\nu, m\delta\nu), \qquad (12)$$

where const. $\tilde{u}\ (n\delta\nu, m\delta\nu)$ is defined as $c(\delta\nu)^2 A_{nm} E(\phi_{nm}/2\pi)$, and A_{nm} and ϕ_{nm} are the magnitude and phase of the transform $\tilde{u}$. Thus

$$W_{nm} \approx A_{nm}; \qquad (13)$$

$$P_{nm} + n \approx \phi_{nm}/2\pi x_0 \delta\nu. \qquad (14)$$

In words, W_{nm} controls magnitude;P_{nm} controls phase. If x_0 is chosen so that $x_0 \delta v$ is an integer M,

$$P_{nm} \approx \phi_{nm}/2\pi M \qquad (15)$$

Equation (15) shows that the lateral displacement is proportional to the phase of the transform, an important relation, which has intuitive meaning as the phase difference of waves originating from two separated Huygens sources (22:4353-4354).

Point Oriented Holograms For point oriented holograms it is required that the complex amplitude to be recorded in the hologram be known for each point on the object (1:311). The complex amplitude of each point on the object is then mapped to each point on the hologram plane. This method has the potential of being very accurate because it uses as its basis the equations of point light propagation as discussed in Appendix A. An exact approximation of an optical hologram can be theoretically achieved using this method if an infinite number of points are used to describe an object and an infinite number of points are allowed in the hologram plane. This of course is not practical so several variations to this method have been developed (1, 18:311-320).

Plane Propagation The methods discussed thus far have been approximations to the 'exact' method of calculating the transmittance values present at the hologram plane. These approximations have been developed because the 'exact' method, which specifies all depth information, is considered computationally expensive. This section is devoted to a discussion of the 'exact' method and an algorithm used to implement this method (1:337-338).

First, consider the reconstruction image shown in fig. 4. The image found in the image plane, z_0, is planar and therefore two-dimensional. It is possible to move the image a distance Δz out of the image plane by building into the CGH an auxiliary lens of the proper focal length. Such an auxiliary lens can be described mathematically as

$$f_e = [1/(z_0 + \Delta z)]^{-1} = [z_0(z_0 + \Delta z)]/\delta z$$

This lens can actually be incorporated into the CGH by multiplying the object Fourier spectrum by the quadratic phase factor

$$exp[i\pi(x^2 + y^2)/\lambda f_e]$$

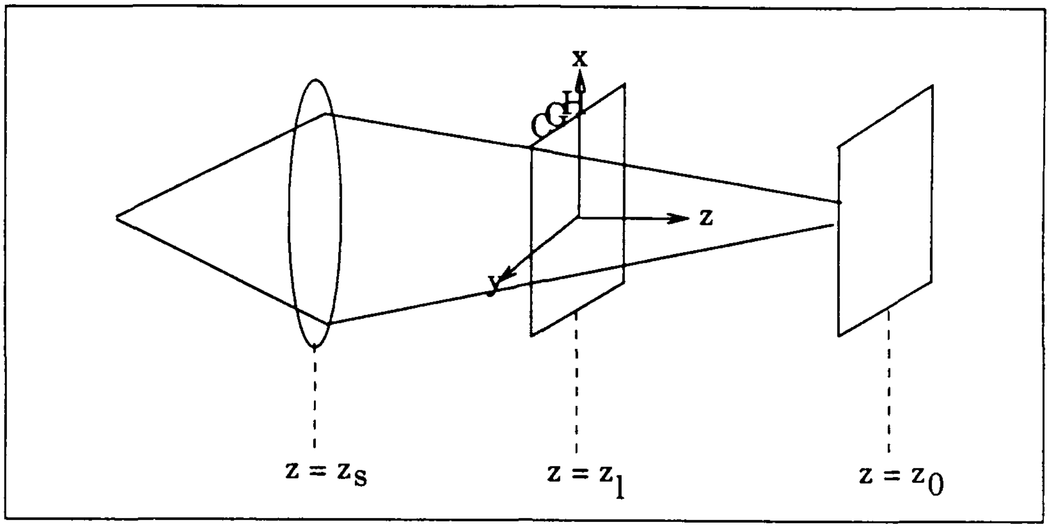

Figure 4. Fourier hologram reconstruction setup (?:291)

Rewriting the quadratic phase factor in terms of the reduced coordinate system yields

$$exp\{i\pi\lambda\Delta z[(z_0 + \Delta z)/z_0](\mu^2 + \nu^2)\}$$

The Fourier spectrum for the displaced object becomes

$$\tilde{u}_d\,(\mu,\nu) = \tilde{u}\,(\mu,\nu)exp[i\pi\lambda\Delta z(\mu^2 + \nu^2)].$$

This equation represents the translation of a plane by Δz along the optical axis.

If an object is described as a collection of planes this translation can be used to propagate each plane from its original position to the hologram plane. As each plane is translated the complex amplitude is summed and multiplied by the transmittance of the plane currently being translated. This method is explained in detail by Ichioka, Izumi, and Suzuki (15). Dallas provides the following analysis of this method.

Because the multiplication of the incident wave by the plane's transmittance is performed in the object domain and the propagating takes place in the Fourier domain, one is constantly transforming from one domain to the other; hence the term ping-pong propagation. If there are N_{PL} planes in the object then $2N_{PL} - 1$ DFTs are performed and $N_{PL} - 1$ transmittance multiplications. One must be very careful that the space-bandwidth product of the hologram, number of resolution cells, is large enough. The image space in the last plane must be surrounded by a zero frame which is large enough to essentially capture waves expanding from previous planes. A good rule of thumb is to multiply the reciprocal of the CGH's f-number at the first plane by the distance from the plane of interest to the final plane. Dividing by the sampling distance in the final plane gives the number of samples in the zero frame. (1:338)

2.2.2 Synthetic Holographic Stereograms Benton has conducted the only recent research aimed at using CGHs for three-dimensional display. Benton's approach is as follows:

> A series of perspective views, corresponding to the expected viewpoints of the audience, are computed by conventional computer-graphic techniques. Each perspective view is then projected with laser light onto a sheet of high resolution film from the angle corresponding to its computed viewpoint, while it is overlapped by a second coherent "reference" beam to produce a holographic exposure that records the direction of the image light. After all the views have been recorded in this way, the hologram is processed and then illuminated so that each view is sent back out in the direction it was projected from. That is, toward its intended viewing location, so that a viewer moving from side to side sees a progression of views as though he of she were moving around an actual object. If the perspectives are accurately computed and registered, the resulting composite image will look like a solid 3-D object. Such a composite or synthetic hologram is termed a "holographic stereogram." It mimics the visual properties of a true hologram, but lacks its enormous information content and interferometric accuracy. (2)

2.3 Conclusion

The potential benefits of computer generated holography are well known and as a result much research has gone into developing viable methods of producing CGHs. Most of the work in this area has been devoted to developing methods applicable to disciplines other than three-dimensional display. There has, however, been some success in developing CGHs for three-dimensional display (18, 1). It appears that all work done so far in the area of three-dimensional CGH has used some form of Fourier analysis. The use of Fourier analysis places a restriction on the size of the object and the size of the resulting hologram. It is possible that the tremendous growth in computing power over the past 20 years will now allow research to begin on methods of developing CGHs without the uses of Fourier analysis.

III. System Development

3.1 Introduction

This chapter details the development of the software system built during this thesis effort. In addition, the different hardware configurations used throughout the thesis effort are also discussed in this chapter.

The software development was divided into two categories. The first was development of the software to calculate the intensity values present in the hologram plane. The second was development of the software to generate the Calma stream format to drive the e-beam lithography machine.

All software was originally written in the C programming language and was run on a Sun 4 and a DEC GPX. Subsequent versions of all software were run on the Cray 2 located at the Supercomputer Center, Kirtland AFB, NM. The unreliability of the Cray C compiler resulted in all software being converted to FORTRAN before reliable results could be obtained from the Cray.

3.2 Intensity Value Software

The Intensity Value Software (IVS) went through three distinct phases of development. Phase one produced Run Length Encoded (RLE) files which allowed the resulting interference patterns to be observed on the screen of a graphics workstation. Phase two produced output in the form of a file containing a value between 0 and 255 for each of the points in the hologram plane. Phase three consisted of translating the phase two program written in C into FORTRAN to allow maximum optimization and vectorization on the Cray 2.

All three phases of the IVS had the same general function. That function was to implement equation 1 as modified in Appendix A, section A.4. This was done by keeping a real and an imaginary part for each element in the array that served as the hologram plane. For each point on the hologram plane, the real and imaginary contribution from each point on the object was calculated using equation 25. After the contribution from each point had been calculated, the contribution from the reference beam was calculated for every point on the hologram plane. At this point the complex conjugate of the resulting complex numbers was taken in order to produce the overall intensity at each point on the hologram plane.

Equation 25 was approximated by using three loops. The outer most loop was executed once for every point on the object. The second loop was executed once for every X value in the array used as the hologram plane. The inner loop was executed once for every Y value in the array used as the hologram plane. The only difference between phase one and phase two IVS was the form in which the resulting intensity values were saved. Phase three was identical to phase two except that phase three was coded in FORTRAN, not C.

3.2.1 Phase One: RLE Output The purpose of phase one IVS was to insure equation 25 had been properly programmed. This was done by calculating the interference patterns for a few very simple objects for which the interference patterns were easily predictable. The hologram plane consisted of a 300 X 300 array of values. The calculated values were normalized by dividing each of the values by the largest value. Since the Sun 4 is capable of providing 256 shades of gray, each of the normalized values was scaled to between 0 and 255. A linear scale was used. The 300 X 300 array of scaled values was then converted into a RLE file for viewing.

The first images to be rendered using phase one software were the interference patterns created by two point light radiators located at distances of 10, 25, 50, and 100 wavelengths from the hologram plane. A photograph of a composite of these four images is shown in figure 5. Clockwise from the upper left, the image shows the interference pattern formed when the separation between the points and the hologram plane is 10, 25, 50, and 100 wavelengths respectively.

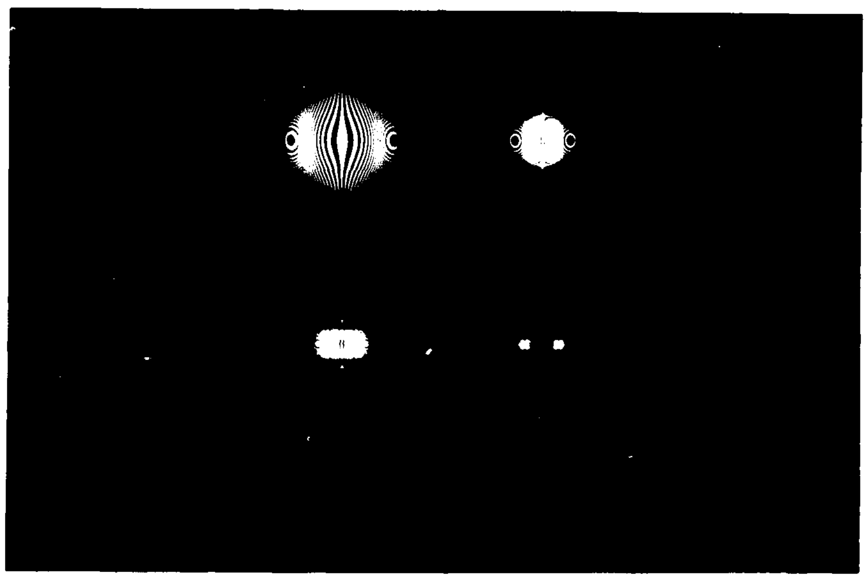

Figure 5. Photograph of first RLE images

Based on these images, it was determined that equation 25 had been properly implemented and work began on phase two IVS.

3.2.2 Phase Two: Binary Output Phase two IVS was identical to phase one IVS in its calculation of the hologram plane intensity values. Only the output method was different. With phase one software, the scaled values between 0 and 255 were used to produce an RLE image. With phase two software, the scaled values were simply output to a file. These scaled values were later used by the plotter software to drive the e-beam lithography machine.

Phase two IVS was uploaded to the Cray 2 to determine what speed improvements could be realized by running on the Cray. It was discovered that the Cray 2 C compiler was unreliable. The major problem found was that the SIN and COS functions do not return correct values for large numbers. Since the use of the SIN and COS functions were vital to the correct operation of the program, it was determined that the final version of the program should be coded in FORTRAN.

3.2.3 Phase Three: Conversion to FORTRAN Phase three IVS was a line by line conversion of phase two IVS into FORTRAN. In phase three IVS the hologram plane was represented by a 36,000 X 36,000 element array. Each element of the array represented a point on the actual hologram plane that was separated by .5 microns from each of its adjacent elements. This means that the array represented a hologram measuring 1.8cm X 1.8cm. Since no computer available had sufficient memory to hold a 36,000 X 36,000 element array, the values in the hologram plane were iteratively calculated in 2000 X 2000 arrays. This resulted in the large array being subdivided into 324 smaller arrays, each of which was labeled by its row and column letter. Figure 6 shows the layout of the large array and the labeling of the subarrays.

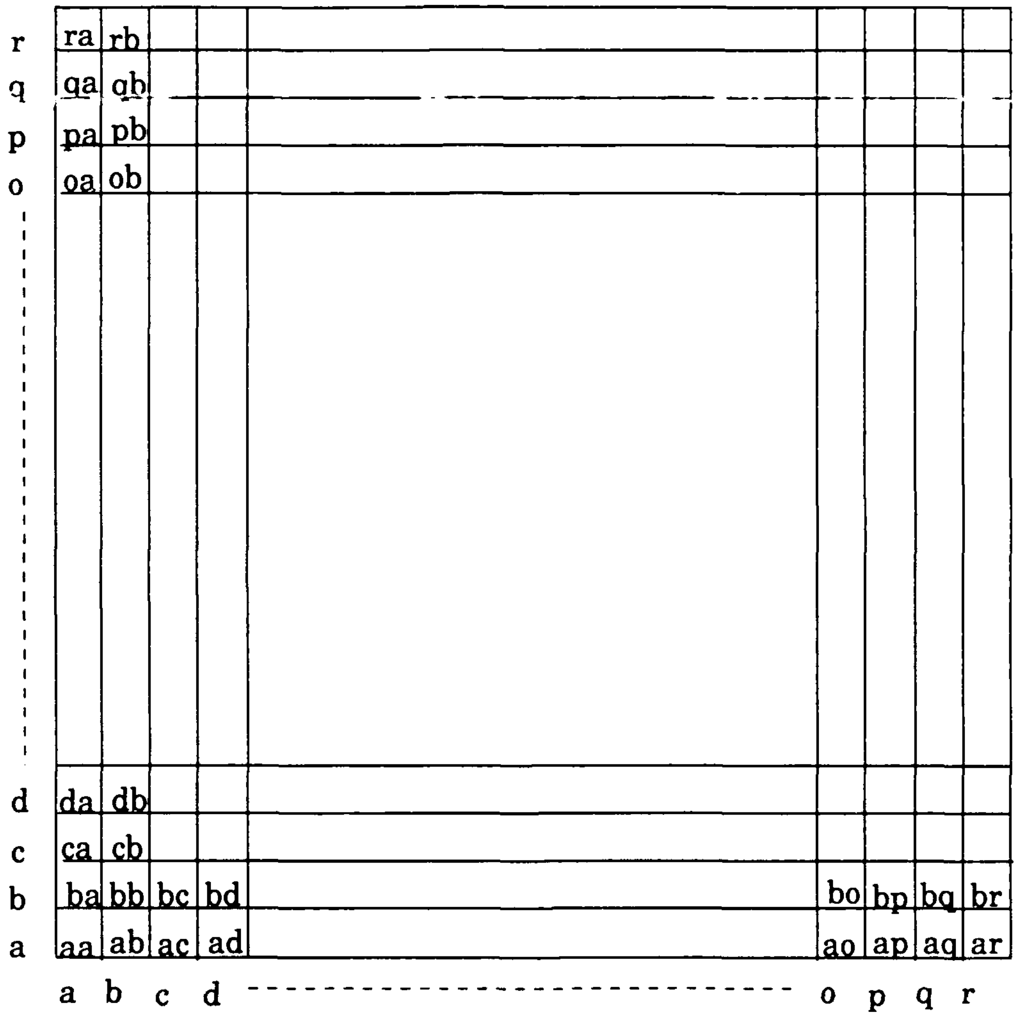

Figure 6. Hologram plane array and its subdivisions

3.3 *Plotter Software*

3.3.1 Calma Electron-Beam Machine Once the IVS was completed and had been executed, it was necessary to write a program to interface the e-beam lithography machine to the data produced by the IVS. The e-beam machine used was produced by General Electric under the name Calma. The e-beam machine is part of a larger computer aided design (CAD) system used to develop very large scale integration (VLSI) circuitry. A computer workstation is used to design the required circuitry. The designed circuit is then converted into the Calma GDSII^{TM} stream format by the workstation. The tape containing the circuit design, in stream format, is then provided to the e-beam lithography machine which etches the designed circuit into a quartz plate. One of the advantages of this system is that a tape containing data in the Calma GDSII^{TM} stream format can be read back into the workstation and viewed on the screen.

The hologram produced by the e-beam machine was actually a quartz plate coated with chrome. The e-beam machine produced the required interference pattern by removing the chrome at only the locations of high intensity. The areas where the chrome has been removed are interference maxima and the areas where the chrome has not been removed are interference minima. See Appendix A, section A.2 for an explanation of interference maxima and minima. The use of an e-beam lithography machine was of significant benefit because no photographic reduction was required. This was due to the submicron writing resolution capability of the e-beam machine.

3.3.2 Intensity Value to Calma Stream Format Interface Software The first plotter software written was an attempt to become familiar with the Calma GDSII^{TM} stream format. The program simply produced the Calma commands to draw two perpendicular lines. The tape containing these commands was taken to WRDC and loaded into the Calma workstation for viewing. Subsequent version of this first program produced the Calma commands to draw increasingly complex patterns, including lines of varying width. Each version was viewed on the Calma workstation to insure the desired results had been attained. This exercise provided the necessary background to develop the actual interface to the e-beam lithography machine.

The software which interfaced the calculated intensity values to the e-beam lithography machine functioned in the following way. The user supplied the name and location of one of the small arrays in figure 6 to the program. As each array was supplied to the program, a threshold was used to convert the intensity values into a binary form. All values above the threshold were considered on and all values below the threshold were considered off. Next, the program began at the bottom of the first column and searched upward until it found the first value above the threshold. The location of this value was stored and the program continued searching upward until a value below the threshold was found. The location previous to the one below the threshold was stored and the program produced the stream commands necessary to produce a line between the two stored locations. If the beginning and ending points were the same, the program looked to the right and left to determine if this point was part of a horizontal line of length greater than one. If it was, the point was not drawn at that time. The program searched every column in this manner until the entire array was complete. At that point the program began with the first row searching for horizontal lines in the same manner. Any vertical line of length one which was skipped in the earlier pass was formed at this time. This two pass approach was used to reduce the total size of the stream format files. If the interference pattern has many horizontal lines in it, a one pass method using a vertical search would be required to write the stream commands for many paths of length one. However, by skipping paths of length one when they are part of a horizontal path reduces the total number of commands which must be produced.

The original plan was to transport the data resulting from the IVS from Kirtland AFB, NM to AFIT. However, transporting 1.3 gigabytes of data turned out to be a difficult task. As a result, the plotter software which was originally written in C, was transported to the Cray 2. Again it was discovered the Cray C compiler was unreliable. For instance, while trying to write out the Calma commands to a disk file it was discovered that the decimal number 10 could not be written out. Instead some other value was written to the file. So this program was also converted from C into FORTRAN.

IV. Results/Conclusion

4.1 Introduction

This chapter contains a discussion of the results and conclusions drawn from this thesis effort. There is a section on the intensity value software, the plotter software, the hologram produced, and the conclusions drawn.

4.2 Results

4.2.1 Intensity Value Software The IVS was used to calculate the object wave front present at a hologram plane. The object, in this case, was a cube. Figure 7 shows the relationship of the cube to the hologram plane. The hologram plane and face A of the cube are parallel to one another and are separated by 50,000 microns. The cube measures 50,000 X 50,000 X 50,000 microns. Beginning in the upper left, moving clockwise, the coordinates of the corners of face A are (-50000,50000,0), (0,50000,0), (0,0,0), and (-50000,0,0). The top, back corner of face B is at coordinate (0,50000,50000) and the bottom, back corner of face B is at coordinate (0,0,50000). All coordinates are given in microns. A total of 55 points were used to define the edges of the cube. Because of the orientation of the hologram plane and its size, only faces A and B of the cube are visible at the hologram plane. As a result, all 55 points defining the cube lie on the 7 edges defining these 2 faces.

A 36,000 X 36,000 element array represented the hologram plane. Each element of the array contained a real and an imaginary part. During the calculation, the real and imaginary contribution from each point on the object was added to the appropriate part of each element of the array. After the intensity values in the first section of the hologram plane were calculated, the IVS was stopped. A program was written to convert the calculated intensity values into and RLE image. This was done in an effort to detect any errors that may have occurred in the calculation. Figure 8

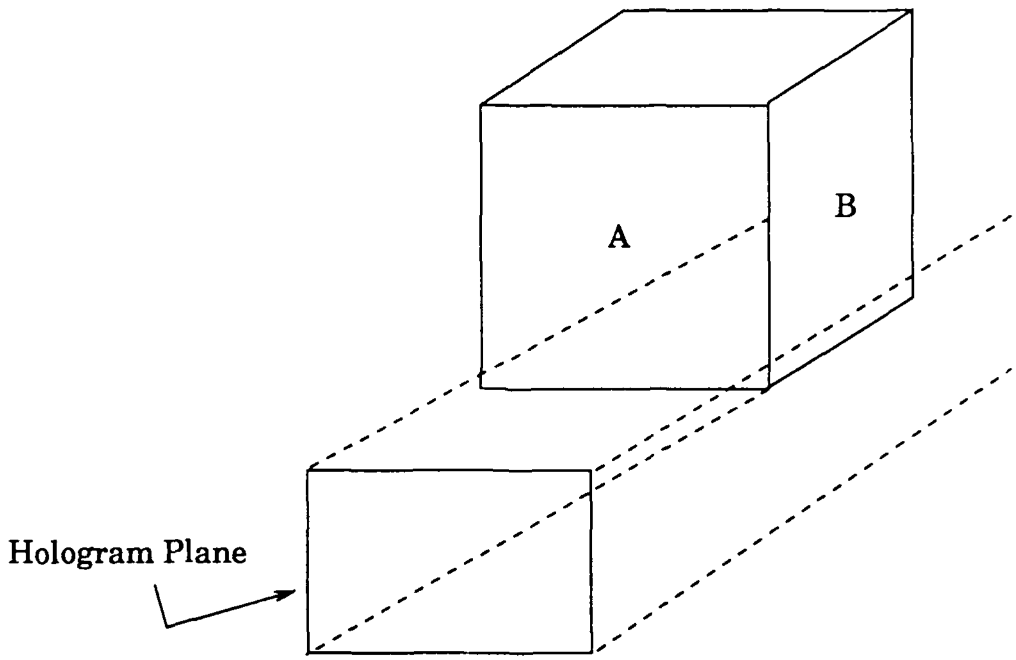

Figure 7. Hologram Plane / Cube Orientation

is a photograph of the resulting RLE image. This image was determined to be correct and the IVS was restarted and allowed to calculate the intensity values for the entire array. After the calculation was complete, the complex conjugate was taken at each element of the array. The resulting real number was then stored as the intensity at that point on the hologram plane. After the normalization and scaling, as discussed in chapter III, section 3.2.1, the resulting one byte for each of the 1,296,000,000 elements of the array was written out to a disk file. Calculation of this 1.3 gigabytes of data took approximately 59.5 hours of Cray 2 CPU time.

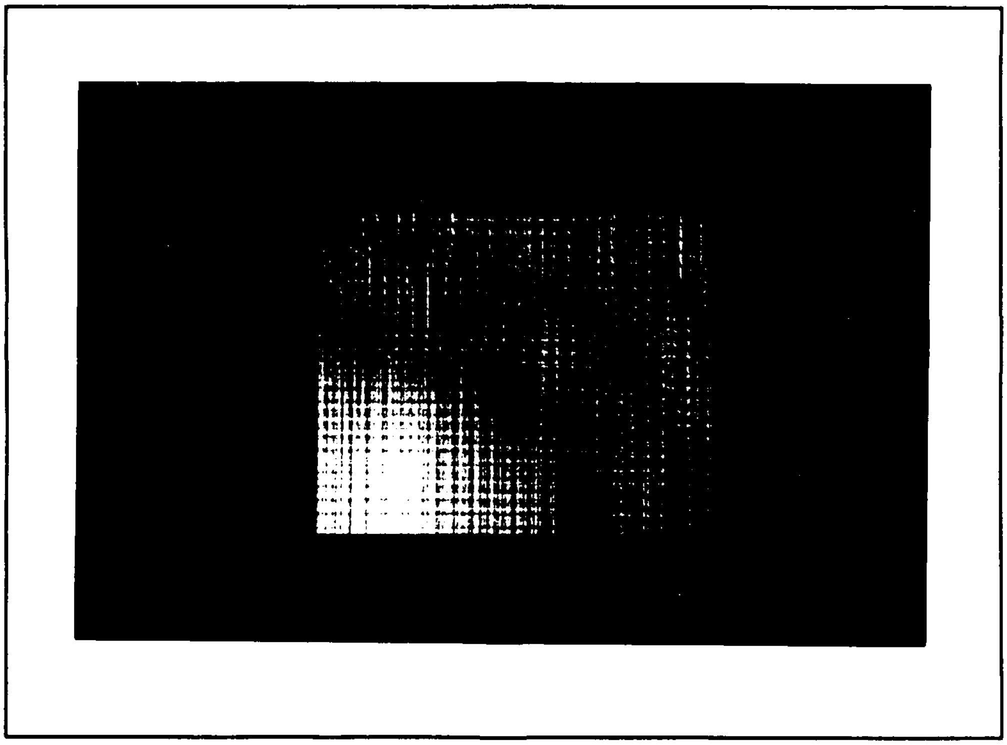

Figure 8. Photograph of Cube Interference Pattern

4.2.2 Plotter Software Each of the small arrays shown in figure 6, which contained the intensity values for that part of the hologram plane, was supplied to the plotter software which created the appropriate Calma commands to produce the interference fringes contained in that array. The calculated intensity values and the required interference fringes were related as follows. Since the e-beam machine had no gray scale capability, a threshold was used to convert numbers into binary form. All values above the threshold were considered on, or of high intensity and all values below the threshold were considered off, or of low intensity. Therefore, if several adjacent elements of the array were above the threshold, an interference maxima was formed there. Likewise, if several adjacent elements of the array were below the threshold, an interference minima was formed there.

Production of all the commands necessary to draw the hologram on the e-beam machine required approximately 5 hours of Cray 2 CPU time.

4.2.3 The Hologram The Calma commands produced by the plotter software did not directly interface to the e-beam machine. Instead, the commands for each section of figure 6 had to be converted to a format which was directly readable by the e-beam machine. No documentation was available for this second format, so the plotter software could not write this format directly. At the time of this writing the conversion process was approximately 50% complete. Each file containing commands for one section of the hologram plane takes approximately 45 minutes to convert. The e-beam machine operators estimate the e-beam machine will require 48 hours to draw the entire pattern once the conversion is complete.

4.3 Conclusions

This thesis effort had three primary purposes as stated in Chapter 1, section 1.2. Each of these purposes is discussed below.

The first and most important purpose of this thesis was to determine if the calculation of the information necessary to make a computer generated hologram of an object was a tractable problem using equation 1. To answer this question the calculation was carried out for a very simple geometric object. Computers in the class that are typically available, such as a Sun 4 or a Vax 11/780, proved to be incapable of making this calculation. This was due to the extremely large numbers used during the calculation. Machines in the class of a Cray were required in order to obtain accurate results. A Cray 2 was used to make the calculation and required almost 60 hours of CPU time to complete the calculation. The result of this calculation was 1.3 gigabytes of data. Most computer facilities do not have 1.3 gigabytes of disk space available to a single user. In light of the excessive time required and the large amount of data produced, it is my opinion that this calculation, as implemented, is not a tractable problem. This method can be used only at facilities with access to the equipment mentioned above.

The second purpose of the thesis was to develop an initial three dimensional computer generated holography capability at AFIT. Fulfillment of this goal required the ability to make the necessary calculations and to convert the calculated intensity values into a form suitable for viewing the three dimensional object. This goal was met. As stated above, a Cray 2 was used to calculate the intensity values. These values were then used to drive a program which produced commands for an e-beam lithography machine. The e-beam lithography machine then drew the interference pattern on a chrome covered quartz plate.

The final purpose of this thesis effort was to begin an investigation into non-Fourier simplifications that could be made to equation 1. This investigation revealed two approaches which might be pursued.

1. The first avenue which could be investigated is an obvious one which is often applied to difficult integral equations. This method is to use Monte Carlo techniques to approximate the solution to the equation. There are multiple forms of these techniques. If one chooses to investigate this avenue further care must be taken to insure the proper techniques are applied to this equation. Several books have been written about Monte Carlo techniques and their applicability to different types of equations. One particularly good book is *The*

Monte Carlo Method by Sobol (21).

2. The second possible area for further research deals with making estimations for part of the equation. As discussed in appendix A, section A.4 equation 1 can be altered to

$$H(X_h, Y_h) = \frac{cos\theta + isin\theta}{((X_o - X_h)^2 + (Y_c - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}}$$

where $\theta = 2\pi\lambda^{-1}((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}$.

Preliminary investigation suggest that an accurate estimate for the value of θ may be obtained by using a step function. In calculating θ, part of the equation is constant, namely $2\pi\lambda^{-1}$. The remainder of the equation is the distance separating a particular point on the object from a particular point on the hologram plane. This distance is calculated by substituting the (X, Y, Z) coordinate of a point on the object for the variables X_o, Y_o, and Z_o respectively. Then the (X, Y, Z) coordinate of a point on the hologram plane is substituted for the variables X_h, Y_h, and Z_h respectively. The appropriate arithmetic is carried out and the result is multiplied by the constant mentioned above. The result is the value of θ. The value of θ could be estimated if the distance part of this calculation could be accurately estimated. This would result in a significant time savings.

Figure 9 shows the geometry of this problem and is a side view. Figure 10 is a front view of the same geometry. As can be seen from figure 9, the distance being calculated is the length of the hypotenuse of a right triangle. In figure 9, when θ is calculated for each point, $P_1 - P_4$, all values in the equation are the same except the Y value in the hologram plane. For each successive calculation of θ, this value is simply incremented by one. If the values $(X_o - X_h)^2, (Y_o - Y_h)^2$, and $(Z_o - Z_h)^2$ are large, incrementing Y_h by one will have very little effect. Therefore, given the distance from a point on the object to a point on the hologram plane, the distance to the next point on the hologram plane should be very nearly the same and predictable. If this is the case, successive values of θ should also be very predictable. If this hypothesis holds true and the value of each successive θ is predictable and changes only a very small amount, it may be possible to estimate the sin and cos of each successive θ.

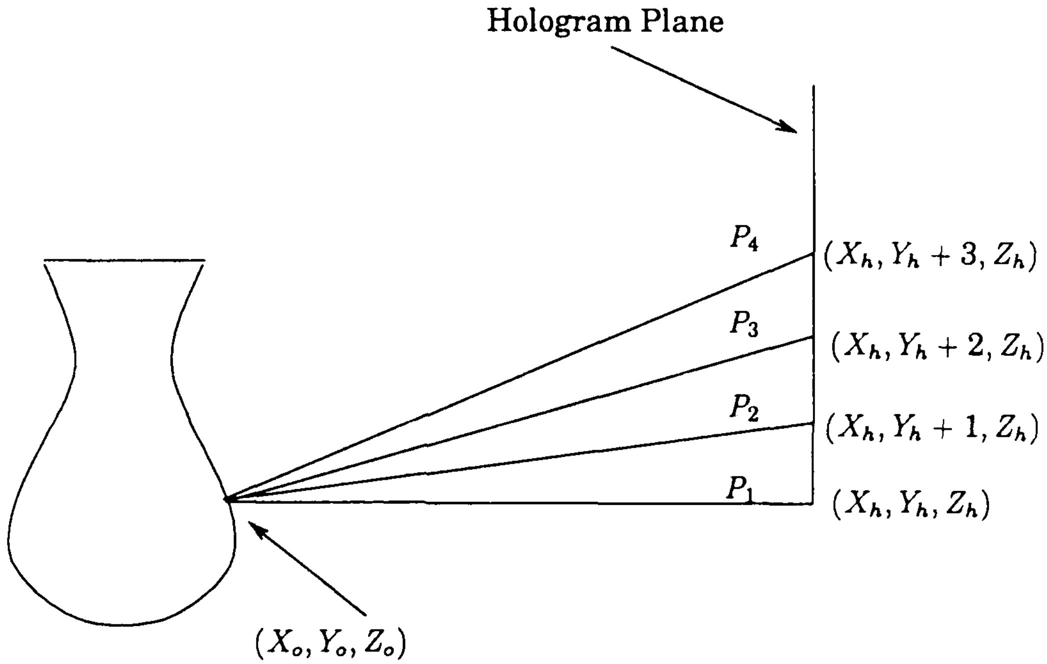

Figure 9. Side View of the Problem Geometry

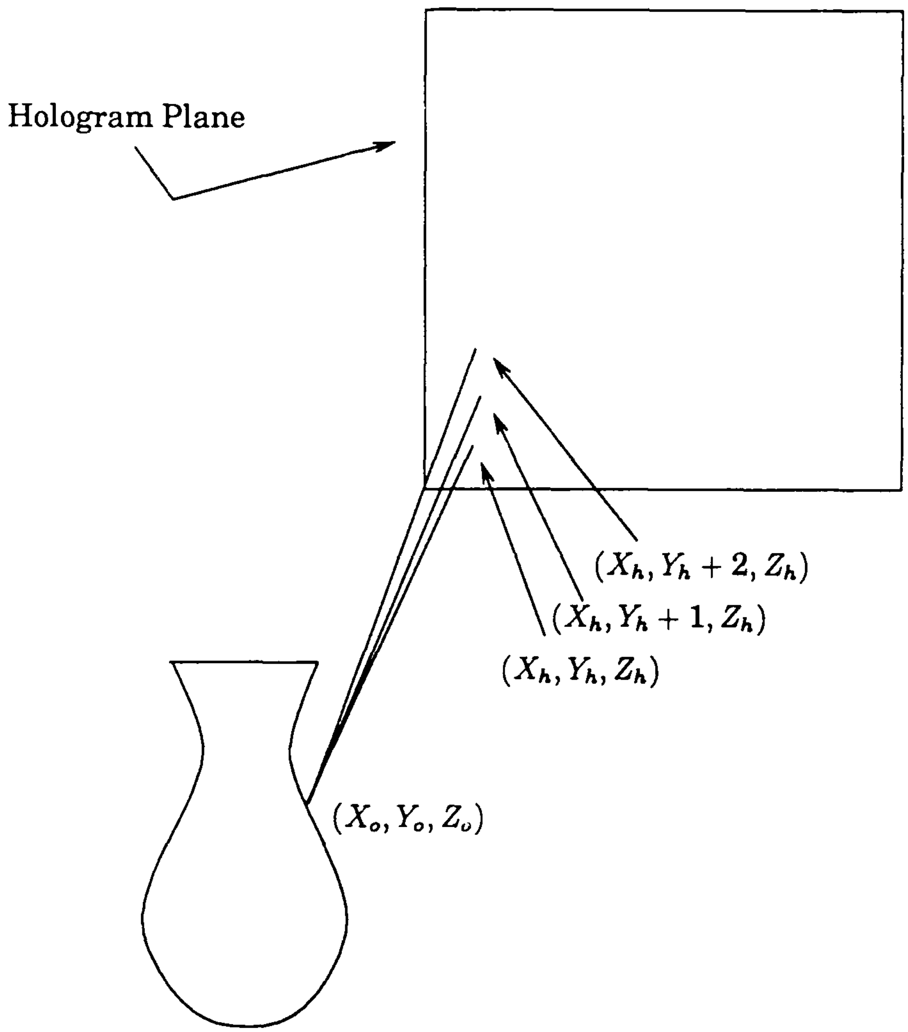

Figure 10. Front View of the Problem Geometry

Appendix A. *Holography*

A.1 Introduction

This appendix contains three parts. Part one is a general discussion of the topic of holography and other relevant topics in optics. It contains historical informat' n as well as a description of the general methods used to produce an optical hologram. Part two contains a presentation of the mathematical theory behind holography. Part three contains a discussion of the calculation of U_o.

Many authors have written books directed specifically at holography and its uses (5, 6, 12). In addition, most contemporary optics books discuss holography. In particular the optics books by Young, Mathieu, and Webb each provide an excellent discussion on the topic of holography (27, 19, 25:120-149,140-141,160-164).

A.2 General Discussion

The term holography is derived from the Greek $\grave{o}\lambda o\varsigma$ = whole or complete and $\gamma\rho\alpha\varphi\epsilon\iota\nu$ = to write or record. Together they mean whole record. Holography can be described as the process by which a whole or complete record of an object is recorded. Usually this recording is stored on a photographic emulsion which is then referred to a *hologram*.

For a complete record of on object to be recorded it is necessary to record both the phase and the amplitude of the light which is scattered by the object. Since a photographic emulsion can record only intensity it is necessary to somehow encode the phase information. This encoding is accomplished by interfering the light diffracted by the object with a coherent reference beam. When these two coherent light sources interfere, alternating bright and dark fringes called interference maxima and minima are formed.

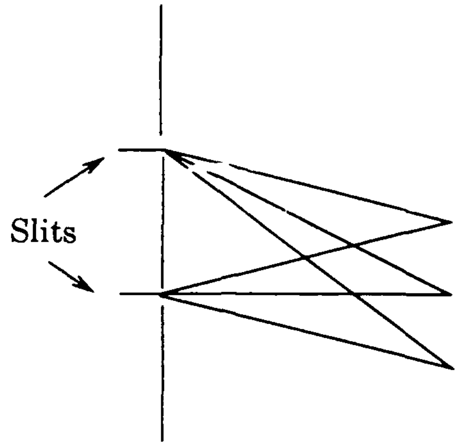

Figure 11. Young's double slit experiment

The classic example of this interference is the double slit experiment first performed by Thomas Young in 1802 (9:63-66). The experiment consists of using two parallel slits and allowing light to pass through the slits and onto a screen. The light from the slits will then produce interference maxima and minima. Young found that the interference maxima and minima occurred in orders. The zeroth order maximum was found in the center of the pattern where the paths from the two slits are equal in length and as a result, the path difference, ς, is zero. The first order maxima is found at the point were the path difference is λ. The second and following order maximas are found where the path difference is an integer multiple of λ. Figure 11 provides an illustration of this. In general, maxima occur when

$$\varsigma = m\lambda, \tag{16}$$

where $m = 0, 1, 2, 3, \ldots$ and is referred to as the order of interference. By convention 1λ is considered to be one trip around the unit circle. That is, the phase angle changes from 0 to 2π radians. Thus, interference maxima will exist whenever the phase angle difference, δ, of the two coherent sources is

$$\delta = 0, 2\pi, 4\pi, 6\pi, \ldots \tag{17}$$

Conversely, interference minima will exist whenever the phase angle difference of the two coherent sources is

$$\delta = \pi, 3\pi, 5\pi, \ldots \tag{18}$$

It is the principle of interference maxima and minima described above that is used to encode the required phase information in a hologram. The fringe pattern of the hologram is formed when two coherent light sources interfere at the hologram plane. One is the reference beam, which is assumed to be a plane wave. In other words, for all points (x, y) in the hologram plane the phase of the reference beam is the same. There is no variation. Since the reference beam position is known, and it is providing a plane wave, the phase is known a priori. The second light source is the object of which a hologram is to be made. Light from the reference beam is split into two beams. One is allowed to proceed directly to the hologram plane as discussed above. The other is directed toward the object. Part of this light reflects off of the object and travels to the hologram plane where it arrives with an unknown phase.

It is at this point the phase information needed to recreate the image of the object is encoded. Interference maxima will be formed at the points where the reference beam and the object beam are exactly in phase. These points form the bright fringes discussed above. Interference minima will be formed where the reference beam and the object beam are exactly π radians out of phase. These points form the dark fringes. Thus, since the reference beam has a known phase at the hologram plane, the bright fringes correspond to locations where the object beam had exactly the same phase. Dark fringes correspond to locations where the phase of the object beam was exactly out of phase with the reference beam. Since the reference beam phase is known a priori, a record now exist which indicates the exact phase of the light from the object.

A.3 Mathematical Theory of Holography

A hologram is a recording of an interference pattern. The interference occurs between the amplitude of two waves U_o and U_r. U_o is the object wave and U_r is the reference wave. As U_o and U_r superimpose the interference fringes are formed and recorded on the photographic emulsion. Since the formation of these fringes is dependent on the phase and amplitude of U_o and U_r, it is crucial that U_o and U_r be kept in a constant phase relationship during the recording. That is, they must be coherent. As they arrive at the hologram plane U_o and U_r combine to give $U_o + U_r$. However, the photographic emulsion can record intensity only. The intensity at the hologram plane is thus given by

$$\begin{aligned} I &= (U_o + U_r)^2 \\ &= (U_o + U_r)(U_o + U_r)^* \\ &= |U_o|^2 + |U_r|^2 + U_o U_r^* + U_o^* U_r \end{aligned} \tag{19}$$

where * indicates the complex conjugate.

When the photographic emulsion is exposed to the intensity I, a transmittance function

$$T \propto I^{-\zeta/2} = (|U_o|^2 + |U_r|^2 + U_o U_r^* + U_o^* U_r)^{-\zeta/2} \tag{20}$$

where ζ is a measure of the emulsion response time.

During reconstruction a wave identical to U_r is made incident on the hologram producing

$$U_{rec} = U_r|U_o|^2 + U_r|U_r|^2 + U_r U_o U_r^* + U_r U_o^* U_r \tag{21}$$

where U_{rec} is the reconstructed wave.

The first term of U_{rec} passes straight through and is unimportant. The second term contributes noise to the scene and is not important. The third term, $U_r U_o U_r^*$, however produces the object wave U_o. The fourth term produces a second, virtual image.

A.4 Calculating U_o

When producing a CGH the problem to be solved is that of calculating the term U_o in the equation discussed in the previous section. U_o is the field present at the hologram plane due to the light reflected by the object. In reality this field is continuous across the hologram plane. Because there exist no recording medium with infinite resolution, and because the field is to be calculated digitally, the problem must be reduced to one of calculating the value of the field at some finite number of points on the hologram plane. This is done by using the following equation:

$$H(X_h, Y_h) = \int \frac{exp[i2\pi\lambda^{-1}((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}]}{((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}} \tag{1}$$

where λ is the wavelength of light, X_o, Y_o, and Z_o are the coordinates of a point on the object, and X_h, Y_h and Z_h are the coordinates of a point on the hologram plane. $H(X_h, Y_h)$ is the value of the field at the point (X, Y) in the hologram plane. The term

$$((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2} \tag{22}$$

is a measure of distance between the current point on the object and the current point on the hologram plane.

Direct computer implementation of equation 1, in this form, is a problem. The problem arises because for this equation to be valid, λ and the distance separating the object and the hologram plane must be measured in the same units. A typical measure for λ is .5 microns (5×10^{-7} meters). Thus, if the object and the hologram plane are at some point separated by six inches in the X direction, six inches in the Y direction and six inches in the Z direction the equation to calculate the partial field at this point on the hologram plane would be

$$
\begin{aligned}
H(X_h, Y_h) &= \frac{exp[i2\pi\lambda^{-1}((25,400)^2 + (25,400)^2 + (25,400)^2)^{1/2}]}{((25,400)^2 + (25,400)^2 + (25,400)^2)^{1/2}} \\
&= \frac{exp[i2\pi\lambda^{-1}(1,935,480,000)^{1/2}]}{(1,935,480,000)} \\
&= \frac{exp[i2\pi\lambda^{-1}(43,994.1)]}{43,994.1} \\
&= \frac{e^{i552,846.2}}{43,994.1}
\end{aligned}
\tag{23}
$$

No digital computer available has the capability to accurately evaluate $e^{i552,846.2}$. Therefore, the form of equation 1 must be altered to allow evaluation of the field U_o at the hologram plane. Euler's theorem states that an equivalent form of $e^{i\theta}$ is $cos\theta + isin\theta$. In this case,

$$\theta = 2\pi\lambda^{-1}((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2} \tag{24}$$

This results in equation 1 becoming

$$H(X_h, Y_h) = \frac{cos\theta + isin\theta}{((X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2)^{1/2}} \tag{25}$$

A.5 Implementation Note

Computer implementation of equations 24 and 25 can be susceptible to numerical accuracy problems. Two things in particular must be avoided. First, one must insure that θ does not become so large that the *sin* and *cos* functions no longer return accurate results. Second, simple numeric overflow must be avoided when the distances become large.

Appendix B. *Fresnel Diffraction*

B.1 Introduction

The equation used in this thesis effort is a general form for scalar diffraction theory. It is not, however, completely general. Furthermore, no current method for making CGHs uses a completely general form of the equation. In all cases, some form of Fresnel diffraction is used to calculate the intensity values.

This appendix contains a brief discussion of the steps that must be taken to transform the general form of the equation into the more restrictive case of Fresnel diffraction. These steps are taken in order to reduce the equation to a form that is more easily manipulated. The book *Introduction to Fourier Optics* by Goodman provides more detail on the subject (11)[57-74].

B.2 Approximation

For the purposes of this thesis effort, diffraction theory was used as follows. Given the properties of light at a given point P_1, determine the properties of that light at the point P_0. Figure 12 illustrates this diffraction geometry. The general form of the equation used to solve this problem is

$$U(x_0, y_0) = \int\int h(x_0, y_0; x_1, y_1) U(x_1, y_1) dx_1 dy_1 \tag{26}$$

where $U(x_0, y_0)$ is the field present at the point $P_0 = (x_0, y_0)$ as a result of the field originating at the point $P_1 = (x_1, y_1)$ and where

$$h(x_0, y_0; x_1, y_1) = \frac{1}{j\lambda} \frac{exp(jkr_{01})}{r_{01}} cos(\bar{n}, \bar{r}_{01}) \tag{27}$$

The first approximation is based on the following assumptions, which refer to figure 12. First, we must assume the distance z between the aperture and the

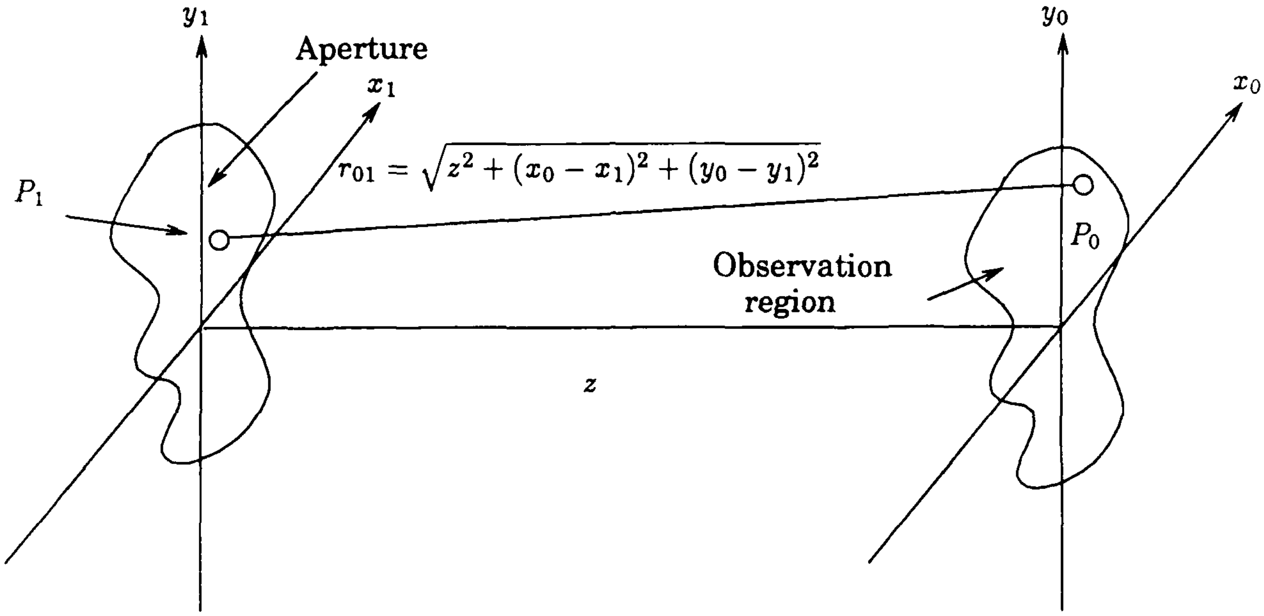

Figure 12. Diffraction Geometry [57]goodman

observation plane is significantly greater than the size of the aperture. Furthermore, we must assume that from the observation plane only a finite area around the z axis is of interest. Finally, we must assume the distance z is significantly greater than the area of the region of interest. For production of 3-D CGHs this means the distance separating the hologram plane and the object must be significantly greater than the size of the object. If these assumptions are true the obliquity factor,

$$cos(\bar{n}, \bar{r}_{01})$$

is approximately one. This is the only approximation made to the equation during this thesis effort. The rest of the equation is calculated.

Under the same conditions, the quantity r_{01} in the denominator of equation 27 is approximately z. This changes the weighting function to

$$h(x_0, y_0; x_1, y_1) \cong \frac{1}{j\lambda z} exp(jkr_{01}) \tag{28}$$

Equation 28 can be further simplified by approximating the quantity r_{01} in the exponent. The distance r_{01} is given by

$$\begin{aligned} r_{01} &= \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2} \qquad (29) \\ &= z\sqrt{1 + (\frac{x_0 - x_1}{z})^2 + (\frac{y_0 - y_1}{z})^2} \qquad (30) \end{aligned}$$

This equation may be approximated by use of binomial expansion of the square root, producing

$$\sqrt{1+b} = 1 + \frac{1}{2}b - \frac{1}{8}b^2 + \cdots \quad |\, b \,| < 1 \tag{31}$$

If the square root in equation 30 is assumed to be adequately approximated by the first two terms of equation 31, the weighting factor can be rewritten as

$$h(x_0, y_0; x_1, y_1) \cong \frac{exp(jkz)}{j\lambda z} exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\} \tag{32}$$

This approximation is known as the *Fresnel approximation.* When all of the above assumptions are valid, the observation plane is said to be in the region of *Fresnel diffraction.* Again, the accuracy of these approximations is governed by the size of the aperture, the size of the viewing region, and the distance z. One test for accuracy that can be used is to require the maximum phase change contributed by the third term of equation 31 to be significantly less than one radian. This condition is met when

$$Z^3 \gg \frac{\pi}{4\lambda}[(x_0 - x_1)^2 + (y_0 - y_1)^2 \tag{33}$$

If the Fresnel approximation is valid, equation 26 can be rewritten such that

$$U(x_0, y_0) = A \int\int U(x_1, y_1) exp[j \frac{k}{2z}(x_1^2 + y_1^2)] exp[-j2\pi\lambda z(x_0 x_1 + y_0 y_1)] dx_1 dy_1 \qquad (34)$$

where $A = \frac{exp(jkz)}{j\lambda z} exp[j \frac{k}{2z}(x_0^2 + y_0^2)]$. In this form the function $U(x_0, y_0)$ can be found by taking the Fourier transform of

$$U(x_1, y_1) exp[j(\frac{k}{2z})(x_1^2 + y_1^2)]$$

and then multiplying the results of the Fourier transform by the amplitude and phase factors.

Appendix C. *Definitions*

Amplitude(A) - is the height of a wave above the wave's average position. Amplitude may range in value between +A and -A. See figure 13.

Intensity(I) - is the amount of energy that flows per unit time across a unit area. The area is defined to be perpendicular to the direction of the flow. Intensity is equal to amplitude squared, $I = A^2$.

Wavelength(λ) - is the distance required for light to travel one full cycle. A full cycle can be measured from peak to peak or from any point on one wave to an equivalent point on the next wave. See figure 13.

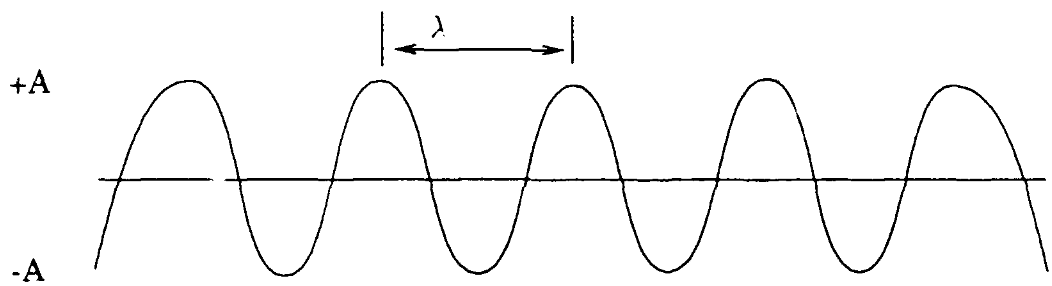

Figure 13. Wavelength and Amplitude

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Vita

Captain Tommy A. Mouser [redacted] In 1984 he graduated from the University of Texas of the Permian Basin with a BS in Computer Science and a BS in Mathematics. In February of the following year he enter Officer Training School(OTS) and was commissioned in April 1985. After OTS he attended Communications Maintenance Officer training at Keesler AFB, Mississippi. In December 1985 he was assigned to Headquarters Air Force Space Command in Colorado Springs, Colorado. In June 1988 he was assigned to the Air Force Institute of Technology.

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19. Cont.

Computer Generated Holograms are used for a variety of purposes. One promising approach is to use them to provide three dimensional views of an object. The methods currently in use to develop computer generated holograms use the Fast Fourier Transform and are not geared toward developing three dimensional images.

This thesis effort developed a method to produce a computer generated hologram by implementing an equation almost identical to that of the general form of scalar diffraction theory. This method will theoretically allow computer generated holograms to me made for a wider range of objects.

The interfence pattern for a cube was calculated using this method. An electron-beam lithography machine was used to transform this pattern to a glass plate.